AC VERSUS DC NETWORKS – CONTROL AND STABILITY THROUGH MODELLING

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A comparison of AC and DC networks primarily in the context of control and stability is undertaken here. The comparison starts with first principles of power system modelling, which leads to the identification of the main limitations in power system operation in AC and DC networks. The possibility for a large range of control approaches is identified. This paper provides preliminary understanding of operation for DC networks, including DC distribution networks.

1. Introduction

Stability has historically been the concern of large AC transmission grids to ensure secure and reliable transport of electricity. The increasing adoption of DC distribution systems is challenging this assumption, as more stability issues may occur closer to the grid edge. This paper seeks to improve the understanding of DC stability issues within the better known framework of AC stability.

The comparison starts with modelling requirements in Section 2 for transient stability analysis of AC networks, and its comparable analysis for DC networks. Section 3 explains stable behaviour. The principle of feedback between output and input is explicitly stated in Section 4. Section 5 explains how synchronisation limits the transfer capacity of AC networks, and the advantage offered by synchronisation in real power control. Section 6 compares power and voltage control between the two types of networks. Lastly, stability analysis is summarized to encourage improvement to methods of understanding stability problems.

2. Modelling and System Representation

Accurate power system modelling of transient phenomena is essential to predict system stability. Models should be of sufficient detail to characterise the main cuases of instability, but not too detailed to reduce simulation complexity. The level of modelling detail required for AC and DC grids is the first cause for difference between these networks. This difference is explained by analysing the simplification of transmission lines for transient studies, which is achieved after defining state models.

State models are a standard representation of differential and algebraic equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}) \qquad (1a)$$
$$0 = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}) \qquad (1b)$$
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}) \qquad (1c)$$

A state model can have multiple inputs $\boldsymbol{u}(t) \in \mathbb{R}^N$**,** which for a network could be the current injection into each node. For the later purposes of control theory, the number of outputs is equal to the number of inputs $\boldsymbol{y}(t) \in \mathbb{R}^N$, which for an electrical network could be the nodal voltages. Internal to the state model are two types of state variables, the differential state variable $\boldsymbol{x}(t) \in \mathbb{R}^M$, and the algebraic state variable $\boldsymbol{w}(t) \in \mathbb{R}^P$. For example, the former can be the current in an inductor, whereas the latter could an AC voltage phasor. The model is characterised by the functions:

$$\boldsymbol{f}: \mathbb{R}^M \times \mathbb{R}^P \times \mathbb{R}^N \to \mathbb{R}^M \qquad (2a)$$
$$\boldsymbol{g}: \mathbb{R}^M \times \mathbb{R}^P \times \mathbb{R}^N \to \mathbb{R}^P \qquad (2b)$$
$$\boldsymbol{h}: \mathbb{R}^M \times \mathbb{R}^P \times \mathbb{R}^N \to \mathbb{R}^N \qquad (2c)$$

Equations (1) and (2) assume all variables are real and vary with time. This requirement is relaxed for AC network phasors that are complex variables.

The advantage of the state model representation is the ease at which two or more smaller state models can be joined together to create a larger state model, where the output of one state model can be the input of another and vice versa. Therefore, to facilitate the description of multiple different state models, a superscript notation in curved brackets is used:

$$S^{(i)} = \left(\boldsymbol{u}^{(i)}, \boldsymbol{w}^{(i)}, \boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)}, \boldsymbol{f}^{(i)}, \boldsymbol{g}^{(i)}, \boldsymbol{h}^{(i)}\right) \qquad (3)$$

where a state model $S^{(i)}$ is a tuple of the variables and functions with an index i to identify the particular state model. The state model that encompasses the connected network of passive electrical components in an AC network is given the index $i = \mathrm{PAC}$; likewise, the equivalent DC network is given the index $i = \mathrm{PDC}$. A power system may have multiple such passive networks connected through active components. Passive components loosely includes resistors, reactors, capacitors, transformers and transmission lines, which are distinguished from the active generators and converters. These definitions lead to the first difference between AC and DC networks.

***Difference 1**: $S^{(\mathrm{PAC})}$ is typically a **non-dynamic** state model, i.e. $\boldsymbol{x}$ and $\boldsymbol{f}$ are not present in $S^{(\mathrm{PAC})}$ when voltage and current are presented as time varying phasors. $S^{(\mathrm{PDC})}$ is a **dynamic** state model.*

Time varying phasors, e.g. for voltage $\tilde{v}(t) \in \mathbb{C}$, are related to the instantaneous quantity, $v(t) \in \mathbb{R}$, by a transformation: $v(t) = \Re\{\sqrt{2}\tilde{v}(t)e^{j\omega_0 t}\}$ where ω_0 is the fundamental frequency of the AC network. The reasoning applied to phasors equally applies to the dq0-transformation of three-phase systems.

The description of the $S^{(\mathrm{PAC})}$ as non-dynamic is consistent with network representation in the equal area criteria. For known voltages at the terminal of the synchronous generator, the current and power transfer through the transmission line is treated as instantaneous. For the equal area criterion, it is the rotational speed of the synchronous generator that is dynamic, which is contained in a separate state model outside of $S^{(\mathrm{PAC})}$.

For $S^{(\mathrm{PDC})}$, converters act quickly to inject current into the DC electrical network to ensure a stable voltage and power transfer. Therefore, with the faster control actions, the dynamics of the DC network become important.

The justification of *Difference 1* can be further refined, but is of more value to explore the implications. *Difference 1* implies a different understanding in the flow of electrical power in each network. For $S^{(\mathrm{PAC})}$, the total power injected into the network has to equal to the total power exported and the losses. Therefore, if a synchronous generator connected to $S^{(\mathrm{PAC})}$ were to lose all mechanical torque imparted to the rotor, then at the time of loss, the electrical power flow into $S^{(\mathrm{PAC})}$ at each node would not instantaneously change. A period of adjustment would begin to shift electrical power generation to the remaining synchronous generators in proportion to their inertia, but the total power injected would remain the same during this initial period except for a small amount of damping from the connected loads and change in power loss.

In an alternative scenario with the loss of a synchronous generator, if its circuit-breaker connecting the synchronous generators to $S^{(\mathrm{PAC})}$ were to open instantaneously, then the state model $S^{(\mathrm{PAC})}$ is inadequate to desribe the AC network voltages and currents until the power input and output are equal again. A electromagnetic transient simulation would be required to model this intervening period.

For $S^{(\mathrm{PDC})}$, if a converter that is supplying power into the DC network were to lose its own source of power, then it can only supply power into $S^{(\mathrm{PDC})}$ for as long as the energy stored within the converter will allow. Once this happens, then the total energy in $S^{(\mathrm{PDC})}$ will start to decrease, which results in an overall reduction in voltages and currents in the network without response from other converters connected to $S^{(\mathrm{PDC})}$. The above discussion is summarized in the second difference.

***Difference 2**: $S^{(\mathrm{PAC})}$ while represented by time varying phasors cannot store energy, but can only transport and dissipate power. $S^{(\mathrm{PDC})}$ can store and release energy.*

3. Stable Operation

This section compares the nature of stable operation for AC and DC networks within a complete power system. A full state model of the complete power system can contain either $S^{(\mathrm{PDC})}$, $S^{(\mathrm{PAC})}$ or many of both, plus the state models of the converters, loads, generators, etc that connect to the electrical networks. It is only once the full state model is formed that stability of the entire power system can be analysed. The entire system is given the index $i = E$, i.e. $S^{(\mathrm{E})}$. The input variables of $S^{(\mathrm{E})}$, i.e. $\boldsymbol{u}^{(\mathrm{E})}$, represent variables of interest whose perturbation is of importance in assessing stability. The number of input variables does not have to equal the number of output variables. The output variables are generally irrelevant and can be omitted from $S^{(\mathrm{E})}$ along with $\boldsymbol{h}^{(E)}$.

Power system stability is the ability for a power system to remain in a stable operating condition once perturbed from a previously stable operating condition. Clearly, this description is a simplification of the types of power system stability characterised in [3], and the formal definitions found in dynamic systems analysis [4]. However, the definition is sufficient for describing the first similarity between AC and DC networks.

Similarity 1*: When assessing transient stability for both $S^{(\mathrm{PDC})}$ and $S^{(\mathrm{PAC})}$ within the same or different state model $S^{(\mathrm{E})}$, the stable and viable operating conditions are asymptotically stable equilibrium points, which are constant solutions with respect to time for $\boldsymbol{x}$ and $\boldsymbol{w}$ in:*

$$0 = \boldsymbol{f}^{(\mathrm{E})}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}) \qquad (8a)$$
$$0 = \boldsymbol{g}^{(\mathrm{E})}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}) \qquad (8b)$$

for a predetermined constant perturbation $\boldsymbol{u}$. Also, it is required that $S^{(\mathrm{PAC})}$ be represented by time varying voltage and current phasors.

The advantage of having stable operating conditions as solutions to (8) is that power system control can be described with steady-state objectives. For example, stable control of a synchronous generator achieves a constant rotor speed with constant voltage phasor angle, constant voltage magnitude, and power injection. However, the concept of stability in *Similarity 1* has important implications on limiting the modelling detail in (8) and does not cover all types of stability problems in power system analysis. These implications are summarized in the remainder of this section.

Firstly, the modelling of $S^{(\mathrm{E})}$ requires the functions $\boldsymbol{f}^{(\mathrm{E})}$ and $\boldsymbol{g}^{(\mathrm{E})}$ to be time-invariant, i.e. not to be direct functions of time, but indirectly through $\boldsymbol{x}, \boldsymbol{w}$ and $\boldsymbol{u}$. Also, the input perturbation $\boldsymbol{u}$ has to reach a steady state. Therefore, stability problems that consider power fluctuations in loads and variable renewable generation requires a different definition of stability.

Secondly, the time invariance of $\boldsymbol{f}^{(\mathrm{E})}$ and $\boldsymbol{g}^{(\mathrm{E})}$ requires the averaging of switching behaviour in converters, or any other devices with regular switching behaviour.

Lastly, if AC network quantities are modelled with instantaneous values instead of phasors, then the stable operating conditions are described as limit cycles instead of equilibrium points, as AC voltages and currents repeat in a sinusoidal cycles. Therefore, *Similarity 1* would be a difference with the instantaneous representation of AC quantities.

4. Primary Principle of Control

AC and DC networks, $S^{(\mathrm{PAC})}$ and $S^{(\mathrm{PDC})}$, are formulated so that the input $\boldsymbol{u}$ contains all current injections for each node in its respective network. Any power system component connected to a network, such as a converter or generator, has its own state model with an output that is the input current injection for either $S^{(\mathrm{PAC})}$ or $S^{(\mathrm{PDC})}$. Also, the connected component has an input that is the node voltage, which is an output from the AC or DC network. Internal to each component is either a controlled current source or a controlled voltage source behind an impedance or resistance. Extended AC and DC network models are created to include either $S^{(\mathrm{PAC})}$ or $S^{(\mathrm{PDC})}$, respectively, and include the parts of the active component state models that contain the controlled voltage and current sources. The extended AC and DC network models are symbolically expressed by $S^{(\mathrm{PAC+})}$ and $S^{(\mathrm{PDC+})}$, where its inputs are the controlled values. For example, a synchronous generator can be modelled by a controlled voltage source behind a synchronous reactance as shown in Fig. 2 for $S^{(\mathrm{PAC+})}$.

Difference 3*: Extended DC networks for every controlled current or voltage source adds* ***one*** *input and* ***one*** *output to $S^{(\mathrm{PDC+})}$, while $S^{(\mathrm{PAC+})}$ expressed with time varying phasors has* ***two*** *inputs and* ***two*** *outputs for each controlled source.*

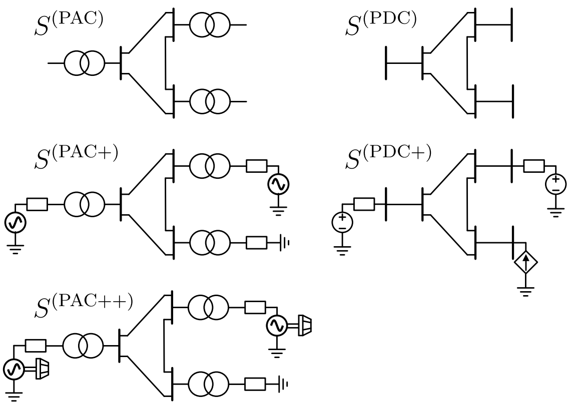

Figure 2: Power system components included in each type of AC and DC network state model.

There are two main implications that result from *Difference 3*. The first implication is common to both AC and DC networks. Because controlled inputs come from connected power system components outside of either $S^{(\mathrm{PAC})}$ or $S^{(\mathrm{PDC})}$, there will always be more state variables $\boldsymbol{x}$ and $\boldsymbol{w}$ than input $\boldsymbol{u}$. Therefore, not every state variable can be controlled. Rather the input variables are to control the output variables.

The second implication is that the extended AC network is twice as complex as the extended DC network of the same topological size, as the extended AC network has twice as many input, state and output variables.

5. Synchronisation

In AC networks, the controlled voltage source of synchronous generators is further controlled by the excitation of the rotor winding and the position of the rotor in the moving reference frame, which rotates at the fundamental frequency ω_0. For inductive AC networks with two or more synchronous generators connected together, the electrical power drawn from each generator adjusts to keep the rotational speed of each generator identical, while retaining a constant phase angle difference between generators to create the necessary voltage drop across network branches to transfer power from generation to load. There is a nature response to restore synchronisation amongst synchronous generators, which does not require control. Furthermore, the electrical power drawn by the generator, including losses, is close to the mechanical power supplied. If any mismatch between the input mechanical power and output electrical power (including losses) does occur, the difference is supplied by the kinetic energy stored in the rotor. The difference in power is common across all synchronous generators, because the rotors are synchronised, and the rotor speed across the whole AC network changes together as kinetic energy is stored and released. This leads to the fourth difference between AC and DC networks.

Difference 4*: The extended AC network including the inertial rotor dynamics of its synchronous generators, call this $S^{(\mathrm{PAC}++)}$, has a common indication of the total power balance between the mechanical power supplied and the electrical power drawn in the derivative of voltage phase anywhere in the network. DC networks can only approximate a common indicator of power balance via external communication.*

Note, power balance for AC networks has to be extended from $S^{(\mathrm{PAC}+)}$ to $S^{(\mathrm{PAC}++)}$ to include the inertial response because AC networks by themselves cannot store energy for the purposes of transient stability analysis. Alternatively, DC networks in $S^{(\mathrm{PDC}+)}$, the power balance directly affects the energy stored in the DC network.

DC networks do not have a common indicator of power balance in and out of the network because a voltage difference across branches is required to transfer power. However, an approximate common indicator can be communicated, but this is unnecessary, as a common indicator is not required for stability.

AC networks do not necessarily require synchronous generators as they can be formed from converters. The control of these converters can be divided into two categories: grid-forming (GFM) or grid-following (GFL) converters. GFL converters controls voltage magnitude and phase for its controllable voltage source in $S^{(\mathrm{PAC}+)}$ to maintain constant real power transfer. Whereas, GFM converters keep voltage phase more fixed to support any power imbalance. GFM converters can have a fully fixed frequency and phase, but it is not recommended to have two or more such converters on the same network unless time is communicated from a central location, such as GPS, with a communicated phase difference amongst sources. Otherwise GFM converters can emulate a synchronous generator with virtual inertia and maintain synchronism through decentralised control.

A common indicator of power balance in AC networks is advantageous in providing a coordinated response without any additional communications infrastructure. However, there is a disadvantage that synchronisation creates, which is the next difference.

Difference 5*: For AC networks where synchronisation is required, power transfer can be constrained to limit the relative phase difference between synchronised sources. DC networks do not have these restrictions.*

The implication of Difference 5 is that DC networks can transfer power over much longer distances for the same power level, which is more important for the selection of HVDC over HVAC.

6. Control of Voltage and Power

Stable operating conditions are solutions to (8) according to *Similarity 1* for $S^{(\mathrm{E})}$. In a similar way, finding steady state solutions for $S^{(\mathrm{PAC}++)}$ and $S^{(\mathrm{PDC}+)}$ with $\boldsymbol{f}^{(\mathrm{PAC}++)} = 0$ and $\boldsymbol{f}^{(\mathrm{PDC}+)} = 0$, respectively, will provide the main philosophies for controlling each network. The controllers that link the input and output variables of $S^{(\mathrm{PAC}++)}$ and $S^{(\mathrm{PDC}+)}$ provide the remaining equations to solve the steady state problem.

Difference 6*: Typically for $S^{(\mathrm{PAC}++)}$ there are two inputs that are often decoupled: the mechanical power, P_M, imparted to the synchronous generator rotor and the DC excitation voltage, V_{EX}. The mechanical power is linked to the output that is derivative of the rotor phase, i.e. frequency. The input excitation voltage is linked to either the output terminal voltage magnitude* *or output reactive power. Typically for $S^{(\mathrm{PDC+})}$, a controlled voltage source behind an impedance, has an input voltage that is linked to the output power injection.*

Difference 6 only indicates the typical links between inputs and outputs. Alternatively, Power System Stabilizers can link excitation voltage with rotor phase and create a coupled control approach to improve rotor damping. Also, some links may not be obvious, as either inputs or outputs are predetermined, e.g. a generator may have a fixed generation, $P_M = 100$ MW. However, not everything can be predetermined, which leads to the second similarity.

Similarity 2*: Not all input P_M for $S^{(\mathrm{PAC++})}$ can be predetermined when finding stable equilibrium points. Also, not all output power injections in $S^{(\mathrm{PDC+})}$ can be predetermined. This is becasue all networks have non-zero power losses that can only be calculated when solving the network equations. Also, in practice, loads are not exactly known and are always changing.*

Furthermore, to *Similarity 2*, *for* $S^{(\mathrm{PAC++})}$ to have a common predetermined frequency, there has to be at least one source of mechanical power only specified by the network equations in $S^{(\mathrm{PAC++})}$. The connected components that provide this P_M are providing a frequency regulation service. Likewise, for $S^{(\mathrm{PDC+})}$ to have a connected component with predetermined voltage, there at least has to be one power injection only determined by the network equations of $S^{(\mathrm{PDC+})}$, which is usually the node with predetermined voltage.

Theoretically, the number of control approaches linking inputs and outputs of $S^{(\mathrm{PAC++})}$ and $S^{(\mathrm{PDC+})}$ is large. Simplicity dictates that linear relationships should be used as much as possible, such as a droop response between P_M and frequency, along with possible deadbands and control limits. Also, external communication infrastructure can allow for different levels of centralised control. Unfortunately, this paper does not have the space to discuss these options.

7. Power System Stability Analysis

Similarity 1 states that stable operating conditions are equilibrium points. However, not all equilibrium points are stable. Usually there is one equilibrium point that is stable and of interest, which minimises losses and voltage drop in the network. This section discusses the tools for analysing the stability of equilibrium points in AC and DC networks.

Small signal stability analysis based on Lyapunov's indirect method can determine the stability of equilibrium points for small perturbations from equilibrium. A linearization of (8) with respect to $\boldsymbol{x}$ and $\boldsymbol{w}$ is required, and can be simplified to give the linear system: $\dot{\boldsymbol{x}} = A\boldsymbol{x}$. It is sufficient for asymptotic stability to have all eignevalues of A with a negative real component, which results in perturbations approaching the equilibrium point with exponential decay. Parameters of the power system can be adjusted to see how the position of the eigenvalues may move from the left half of the complex plane to instability on the right.

Large signal stability that is performed by time-domain simulations assesses how large disturbances affect power systems over a longer period.

Tools for large and small signal stability analysis have found wide application in industry, and are offered by many commercial software packages. These tools are necessary components of system studies for planning major projects. However, these tools in themselves cannot offer a full understanding to design and plan AC and DC systems. Therefore, other methods of stability analysis, such as impedance analysis methods and passivity based methods, are gaining importance in providing design principles [5].

Impedance analysis is based on assessing the stability of two electrical systems once connected. One system is typically the electrical network viewed from the point of connection of a device that is the second system. Firstly, it is necessary to demonstrate that each individual system is stable. For example, that the grid is stable for constant current, and that the device is stable for constant voltage, or vice versa. Both the grid and device have an apparent impedance or admittance from the point of view of the common connection. The second step is demonstrating that the iteractions between the two systems are stable, which is achieved by determining the position of the poles of the transfer function:

$$\frac{1}{1 + Z_1(s)/Z_2(s)}$$

where $Z_1(s)$ and $Z_2(s)$ are the apparent terminal impedances of each system. The ratio of the two impedances is the minor loop gain, where the Nyquist stability criterion can be applied to determine if any right hand side poles exist. Impedance analysis requires linearisation of system models just as small signal stability requires. The advantage of impedance analysis is that a full system model of the device is not required, but can be represented by its frequency response and stability assessed by the Nyquist stability criterion. Furthermore, impedance analysis can indicate how the frequency response can be modified to improve stability margins.

Passivity analysis is applied to state models that include an input and output of equal number. A passive system has the appearence of a resistor over all frequencies, e.g. the input voltage of the state model is multipled by the output current, which always has a positive power into the system. A passive system damps oscillations as energy is dissipated, which results in stability.

The utility of passivity analysis is that two passive systems connected in a negative feedback is also passive. This feedback can represent the electrical connection of the network to a device. The output voltage from the network is the input into the device, and the current output of the device is the input into the network. The challenge of passivity analysis is that electrical loads under constant power control appear as negative resistive loads from a small signal linearisation perspective. Therefore, techniques have been adopted to exchange the level of passivity between the two systems to ensure an overall passive network.

Each type of stability analysis can be applied to both AC and DC systems, and the combination of these systems. The main difference in stability analysis between AC and DC networks is that AC networks that are composed of synchronous generation in the transmission grid only require a stability assessment at the transmission level, where the dynamic properties of loads are aggregated. Operational constraints ensure adequate reserves are available for contingencies. Planning and testing ensure that individual generators are stable during their most stressed operation, which is when the generator is electrically isolated from the rest of the network, but still connected to nearby load. Also, transient analysis in system studies ensures that the complete system retains synchronism and voltage stability.

For any power system dominated by power electronic converters, for both AC and DC networks and at transmission and distribution level, the stability of the system has to be verified at each voltage level. This creates a greater challenge as more parts of the network have to be assessed for stability.

8. Conclusion

A high level view is offered on the differences and similarities of how AC and DC networks are controlled and stability is assessed. Both AC and DC networks have similar objectives to control voltage and power, but the slower response time of synchronous generators and their ability to maintain synchronism is a significant cause of differences. This paper provides an opportunity to reassess assumptions about how AC and DC networks operate and how that might change as more converter connected resources become prevalent. A number of generalisations have been made in this process so that attention can be given to the primary principles.

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